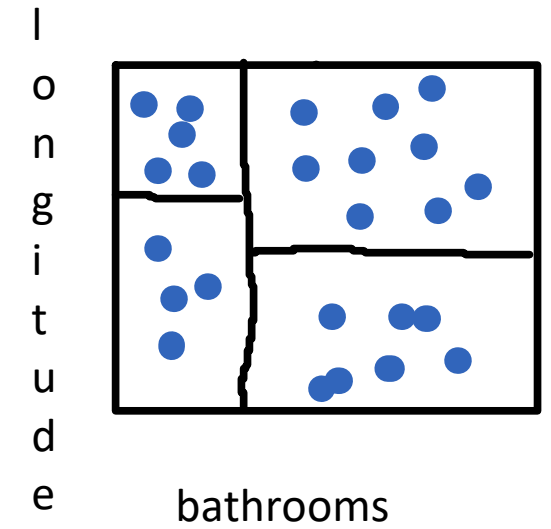
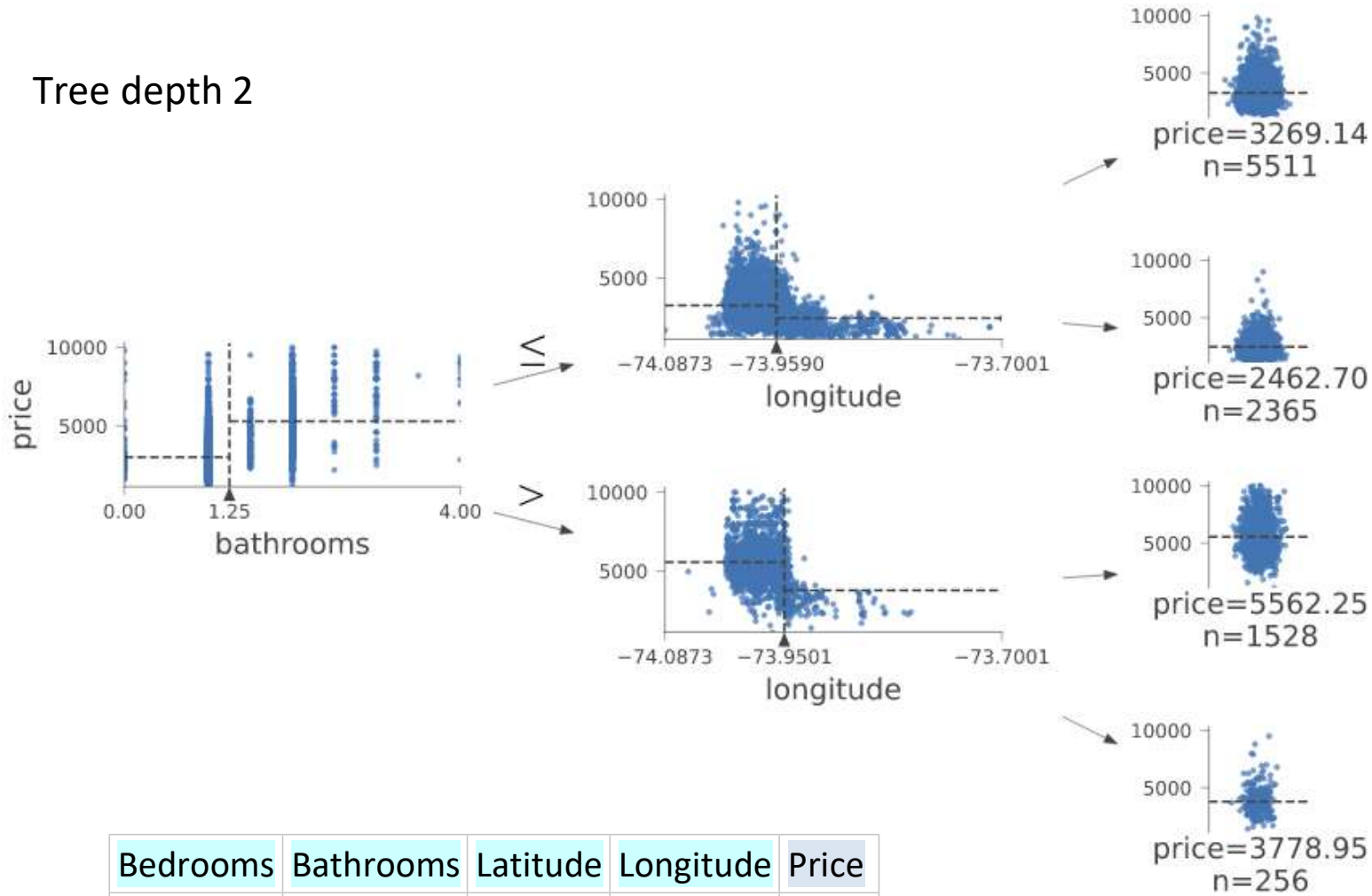
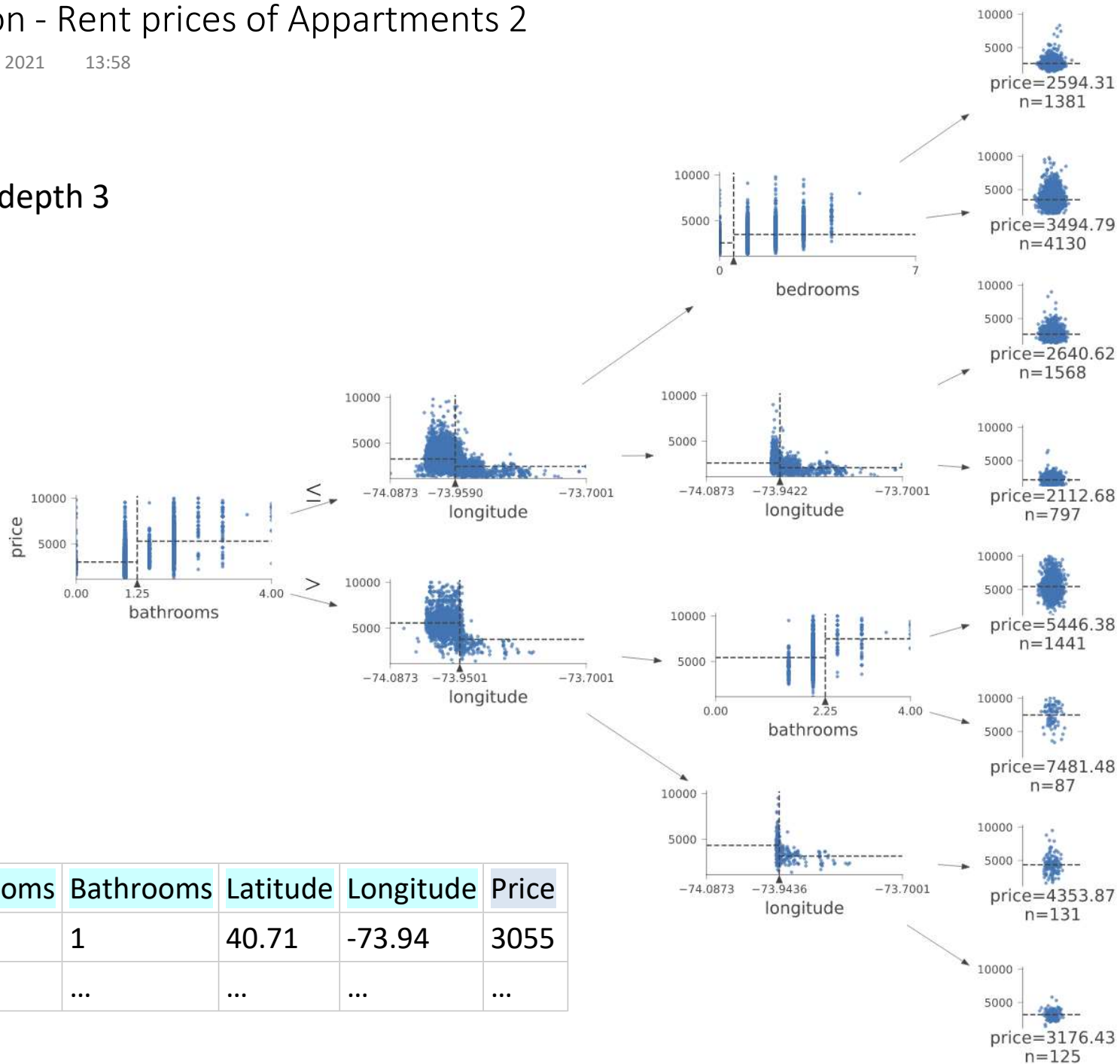


Tree depth 2



Bedrooms	Bathrooms	Latitude	Longitude	Price
1	1	40.71	-73.94	3055
...

Tree depth 3



Bedrooms	Bathrooms	Latitude	Longitude	Price
1	1	40.71	-73.94	3055
...

Feature space:

- 2 features: x_1, x_2 : plane
- 3 features: x_1, x_2, x_3 : cube
- n features: x_1, \dots, x_n : n -dimensional cube

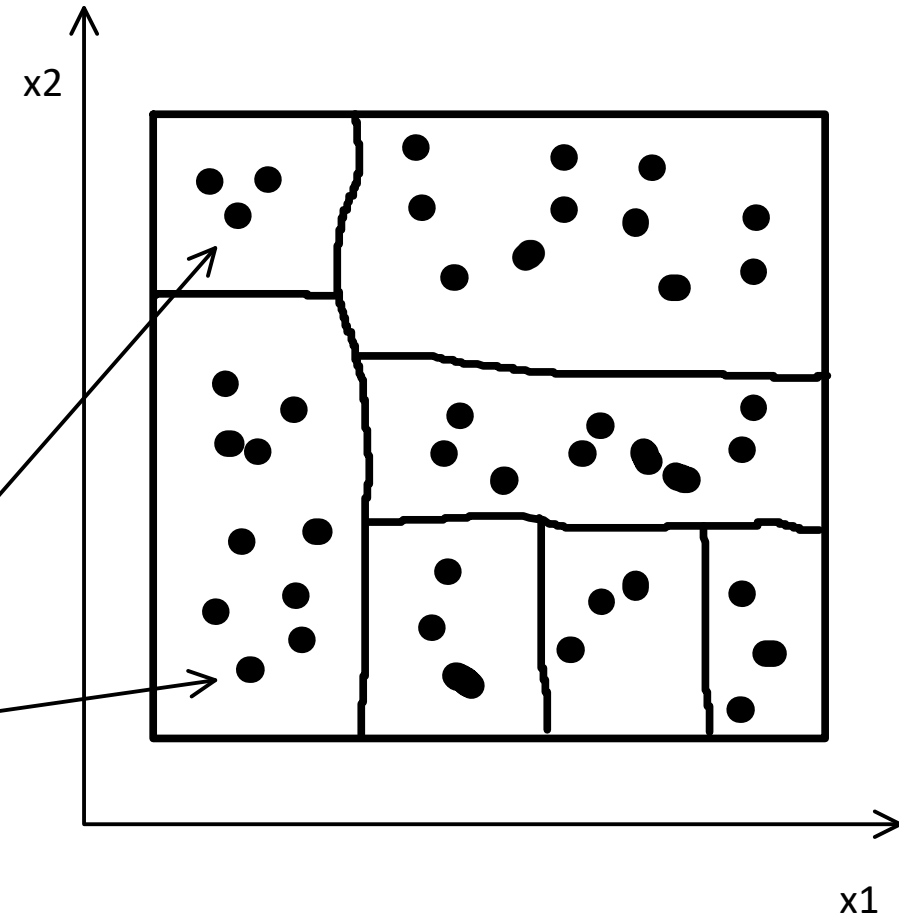
Find rectangular non-overlapping decomposition of space

- Leads to n -dimensional sub-cubes (regions)
- Each region should be as "uniform" as possible
- Average value of all records in region (regression)
- Majority vote in region (classification)

Simplification for presentation purposes

- Only two features: x_1, x_2
- Regions are rectangles

Instances



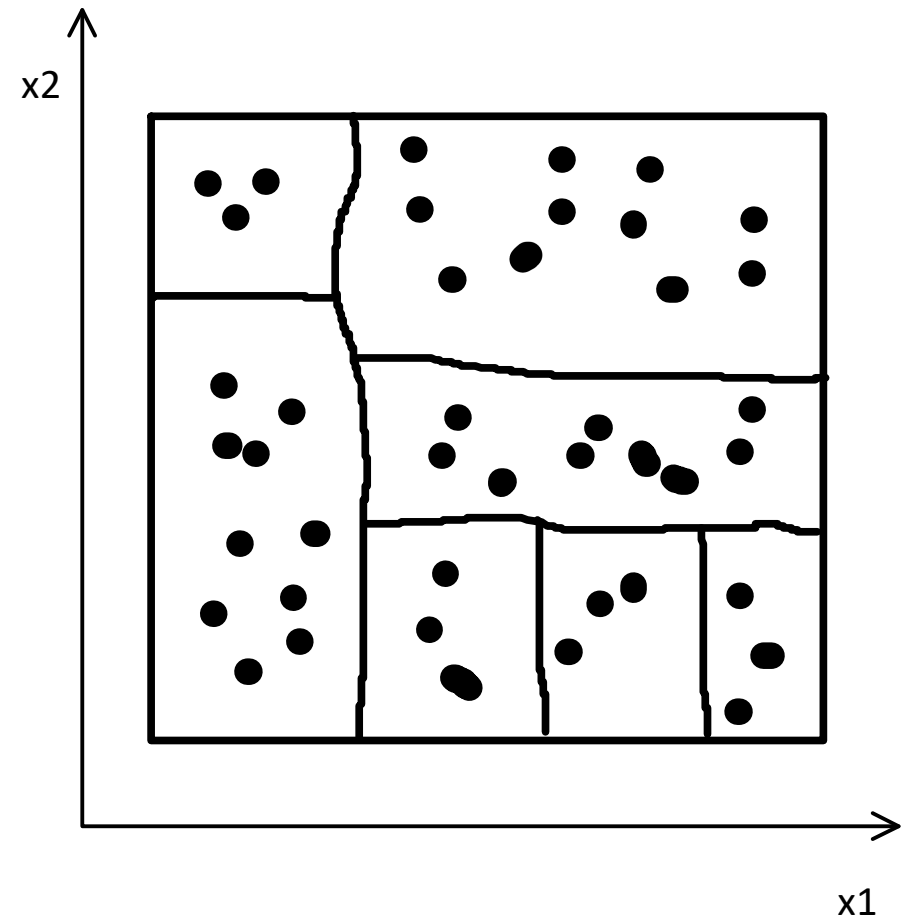
Finding an optimal decomposition automatically is infeasible (combinatorial explosion)

General idea of decision trees:

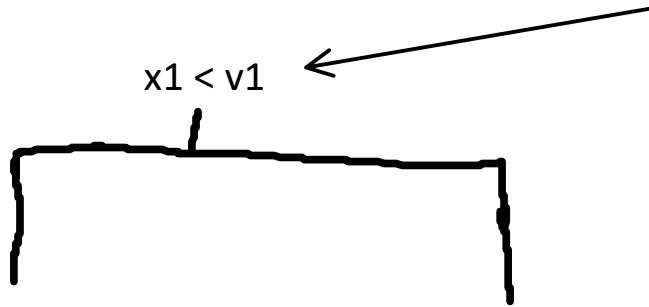
- Greedy decomposition of feature space by recursive binary splitting
- Stop splitting according to criterion, e.g.
 - minimal numbers of instances in region
 - max depth of tree
 - minimal performance gain

Finding splits

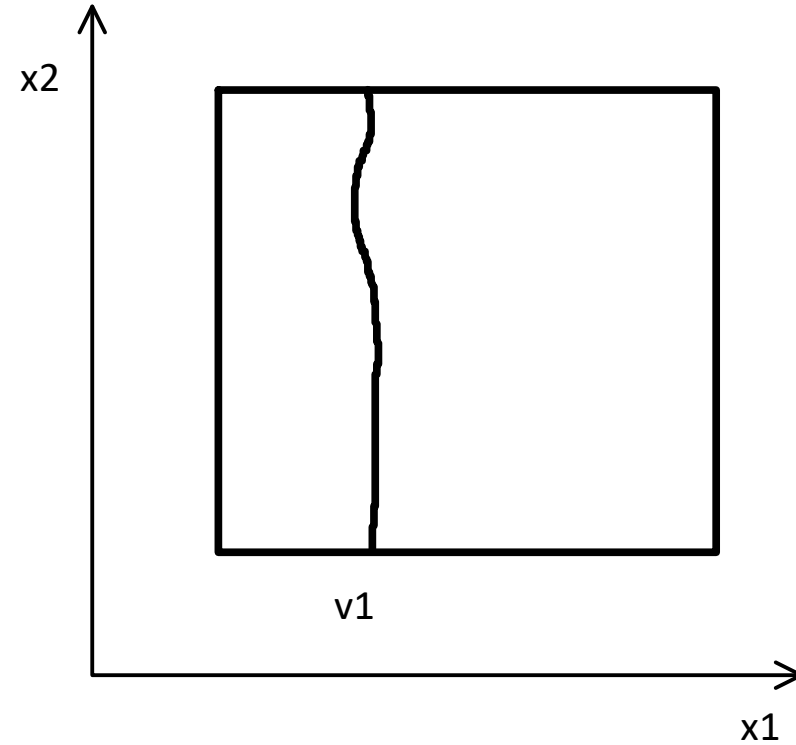
- Search through all features
- For each feature: consider all split values
- Take "best" (feature, splitval) combination



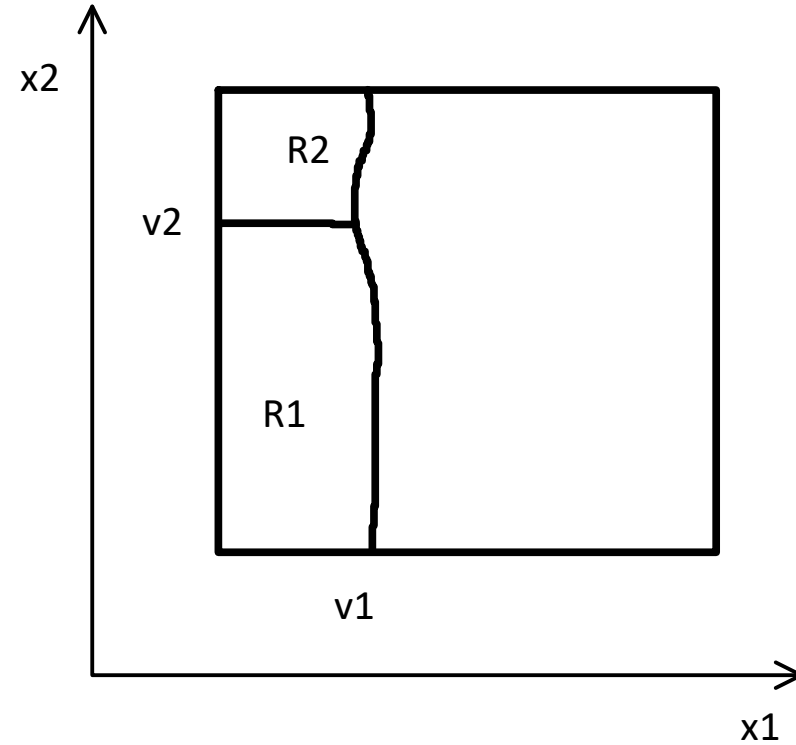
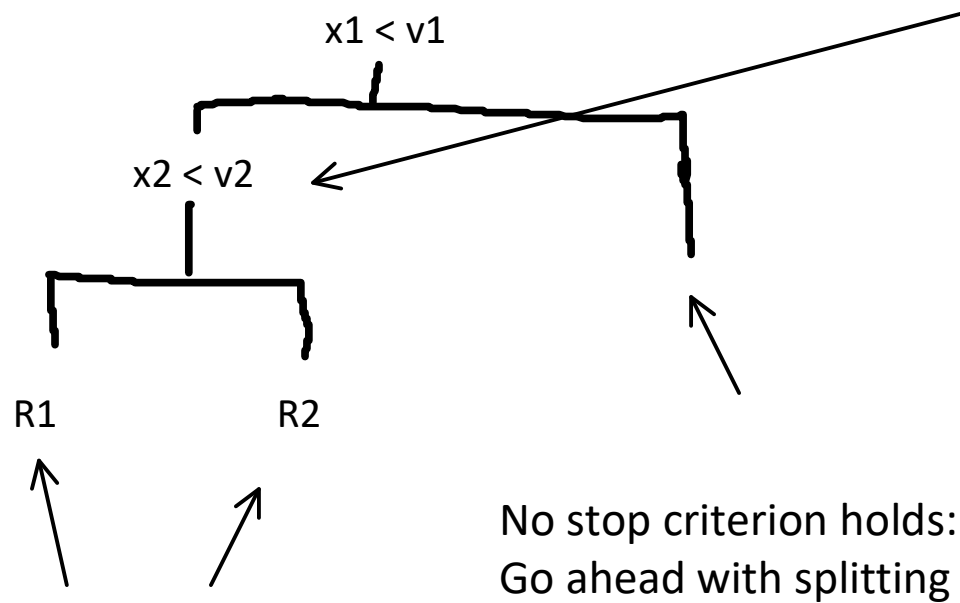
split feature	split value
x1	v1



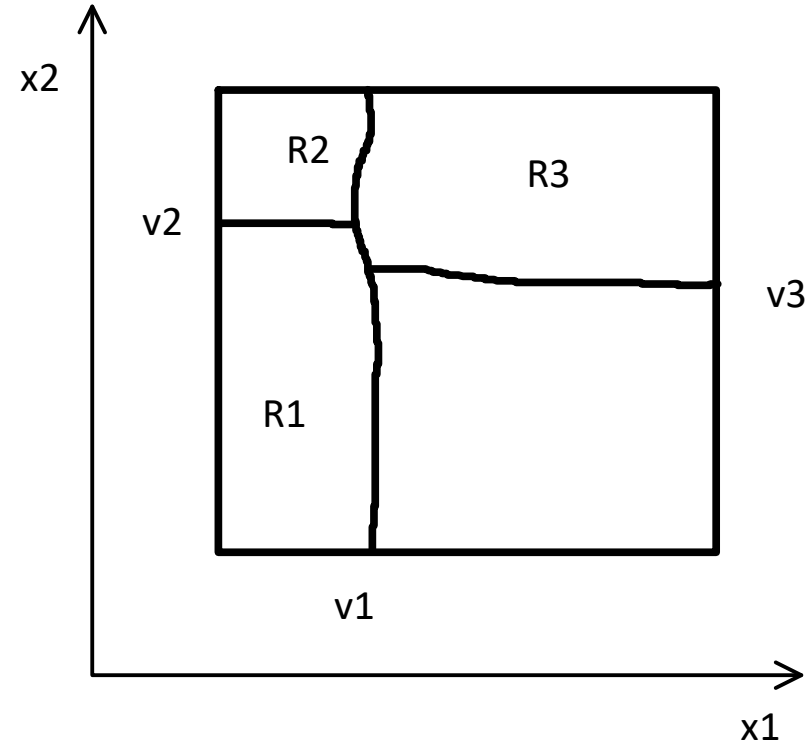
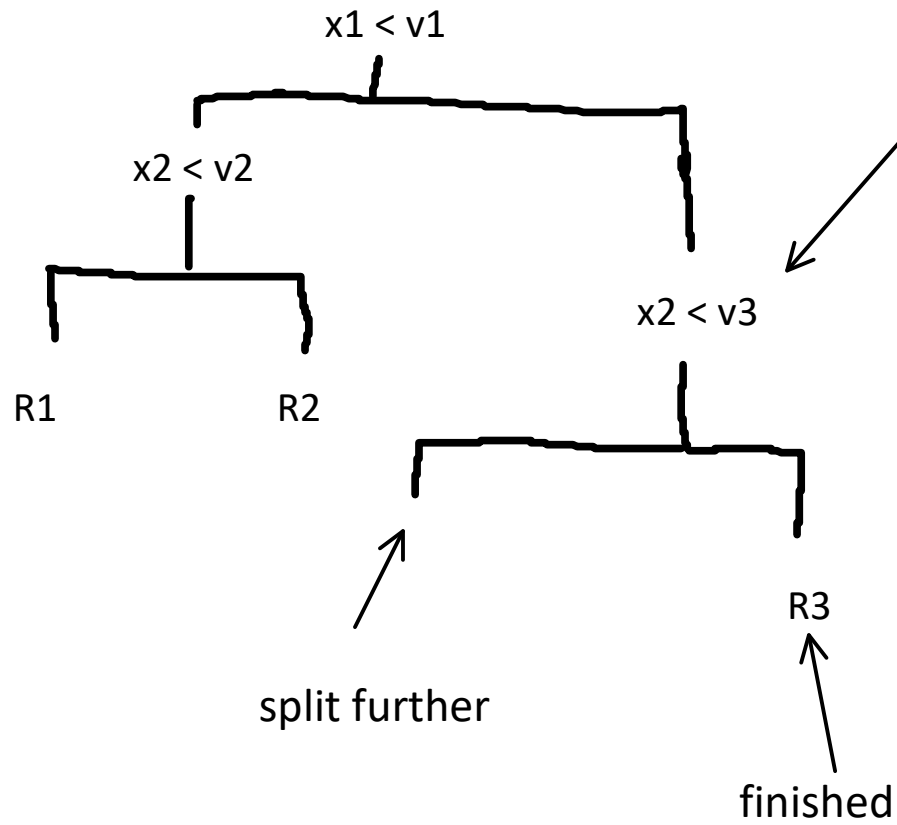
No stop criterion holds:
Go ahead with splitting

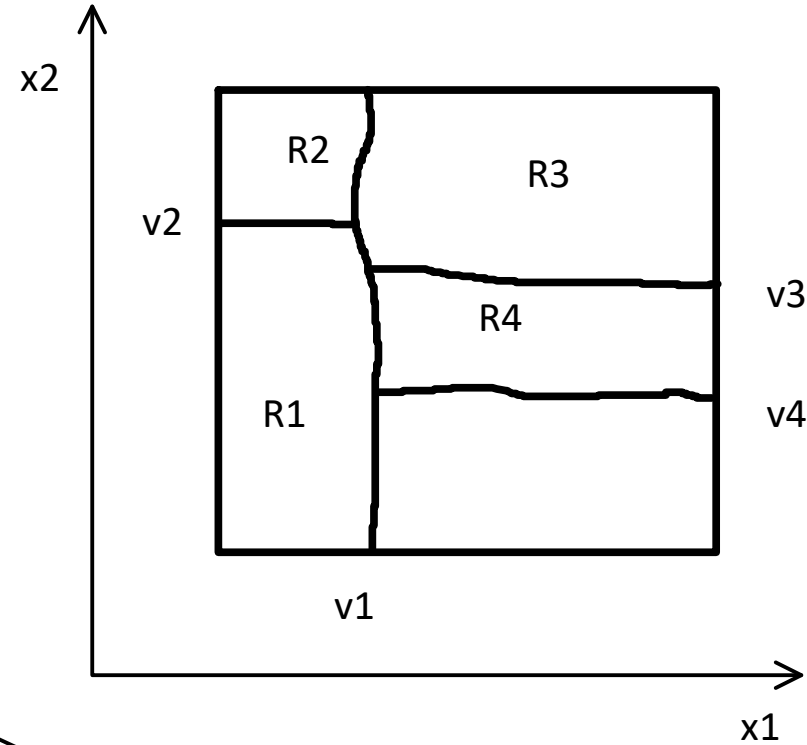
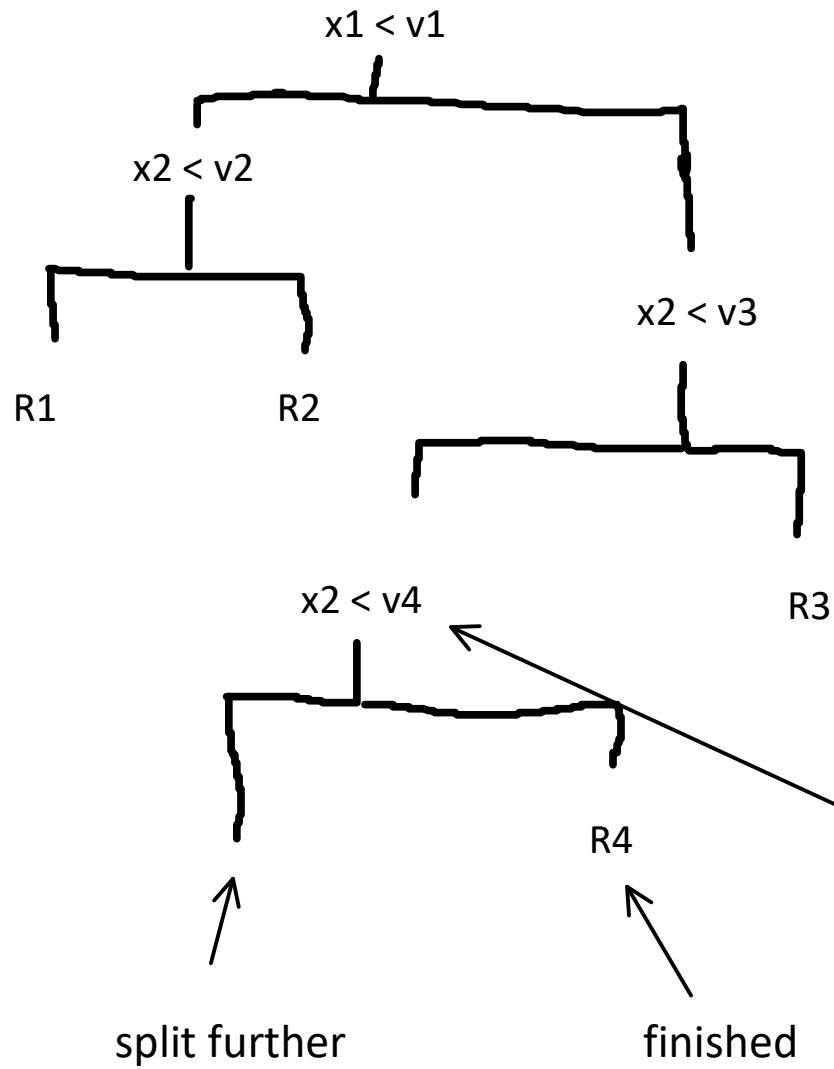


split feature	split value
x2	v2

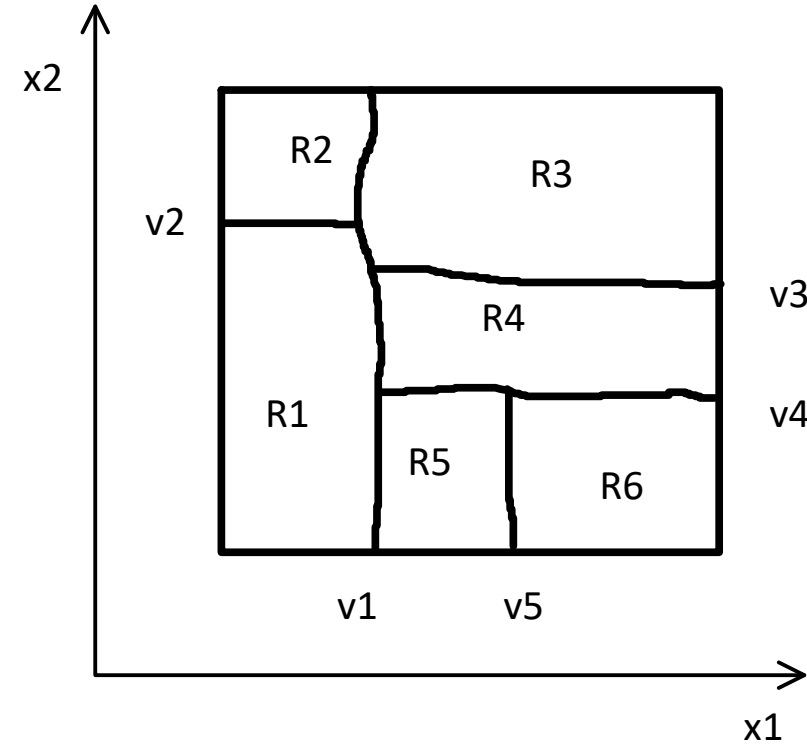
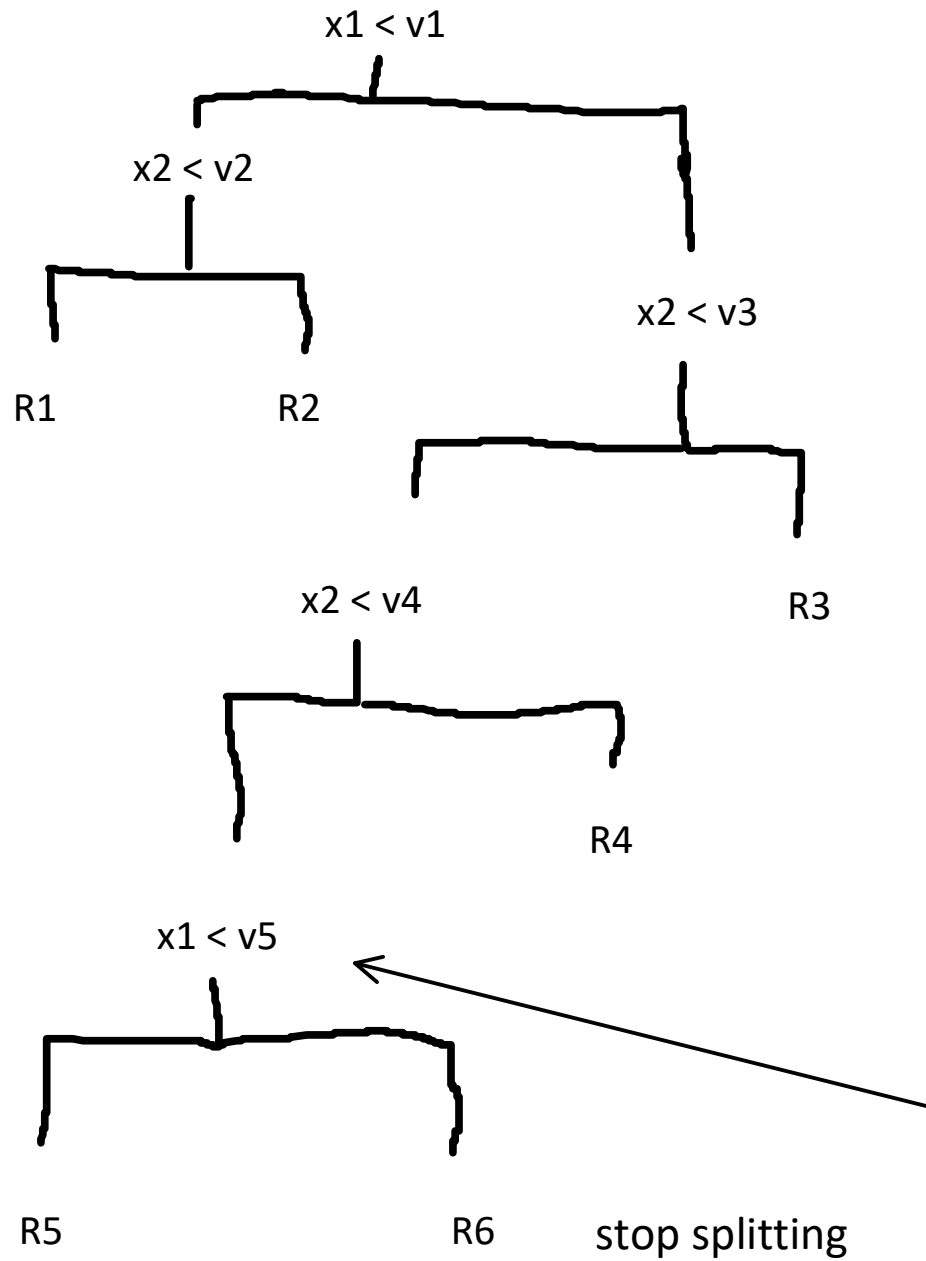


split feature	split value
x2	v3



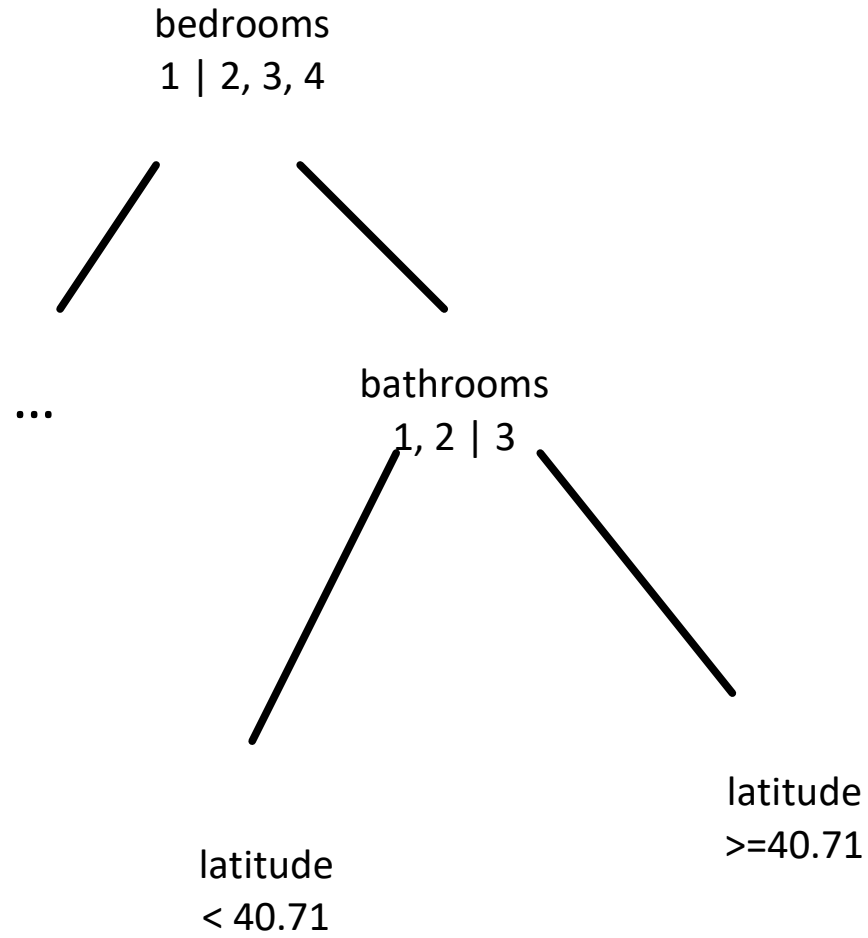


split feature	split value
x2	v4



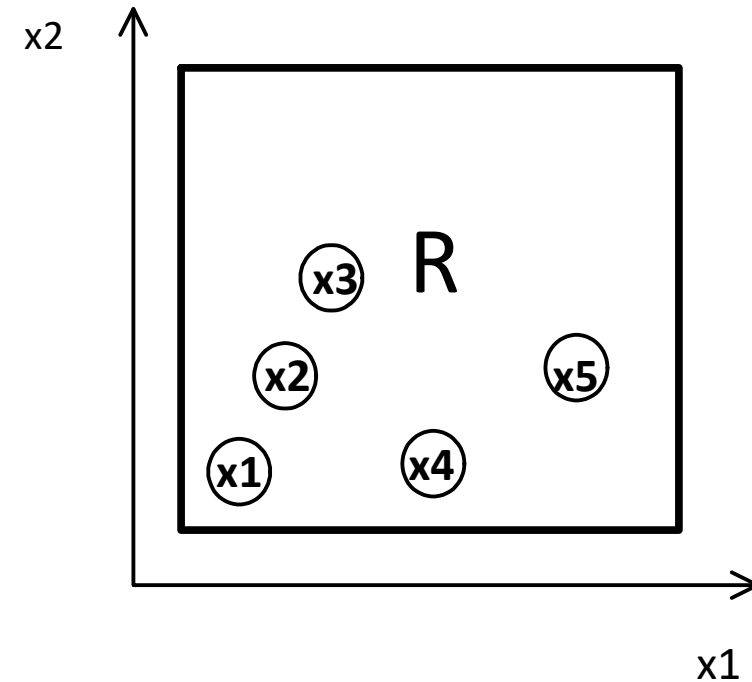
split feature	split value
x1	v5

Split on best combination
of feature and split point



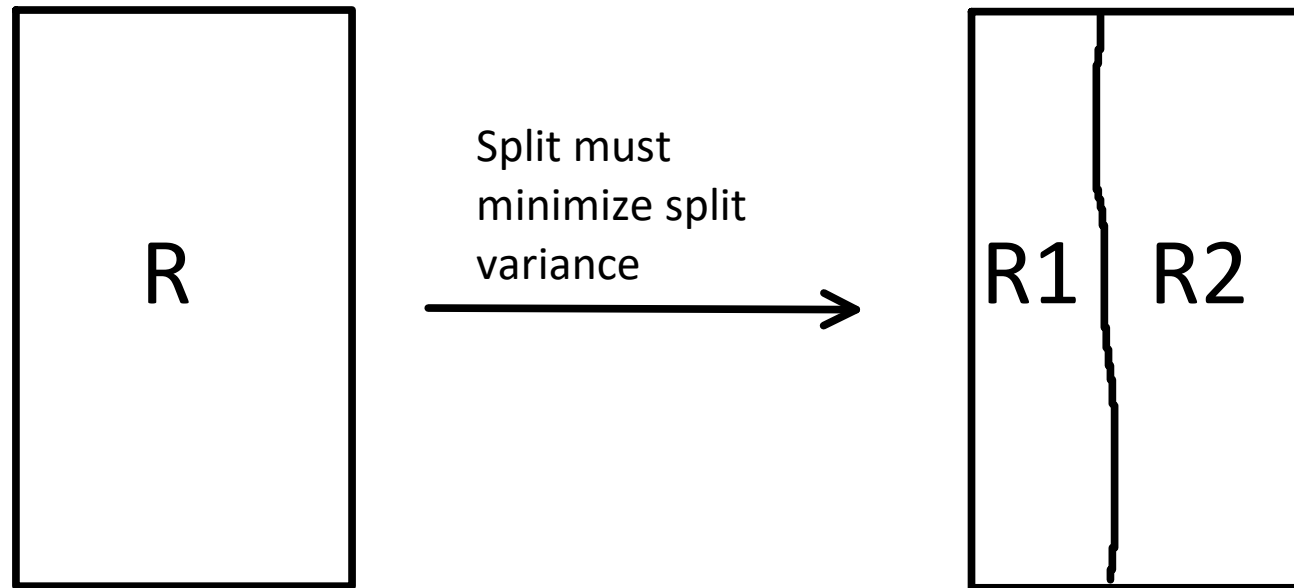
$$\text{MSE}(R) = \frac{1}{\text{count}(y_i)} \sum_{\mathbf{x}_i \in R} (y_i - \text{avg}(y_i))^2$$

	x1	x2	y
x1	1	1	20
x2	1	2	30
x3	2	3	20
x4	3	1	10
x5	4	2	20



$$\text{avg}(y_i) = 20$$

$$\begin{aligned} \text{MSE}(R) &= \frac{1}{5} ((20 - 20)^2 + (30 - 20)^2 + (20 - 20)^2 + (10 - 20)^2 + (20 - 20)^2) \\ &= \frac{1}{5} (0 + 100 + 0 + 100 + 0) \\ &= \frac{1}{5} * 200 \\ &= 40 \end{aligned}$$



$$\text{Split Variance} = \frac{1}{\text{count}(R_1)} \text{MSE}(R_1) + \frac{1}{\text{count}(R_2)} \text{MSE}(R_2)$$

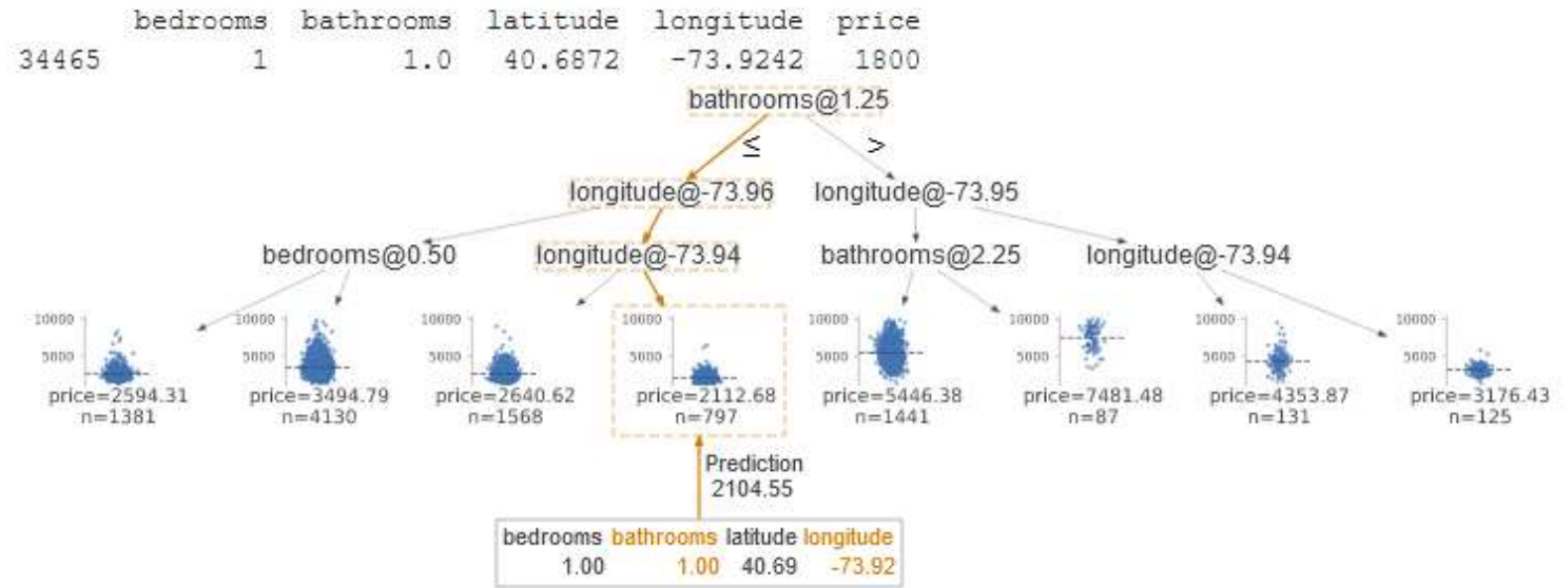
Donnerstag, 1. Juli 2021 15:33



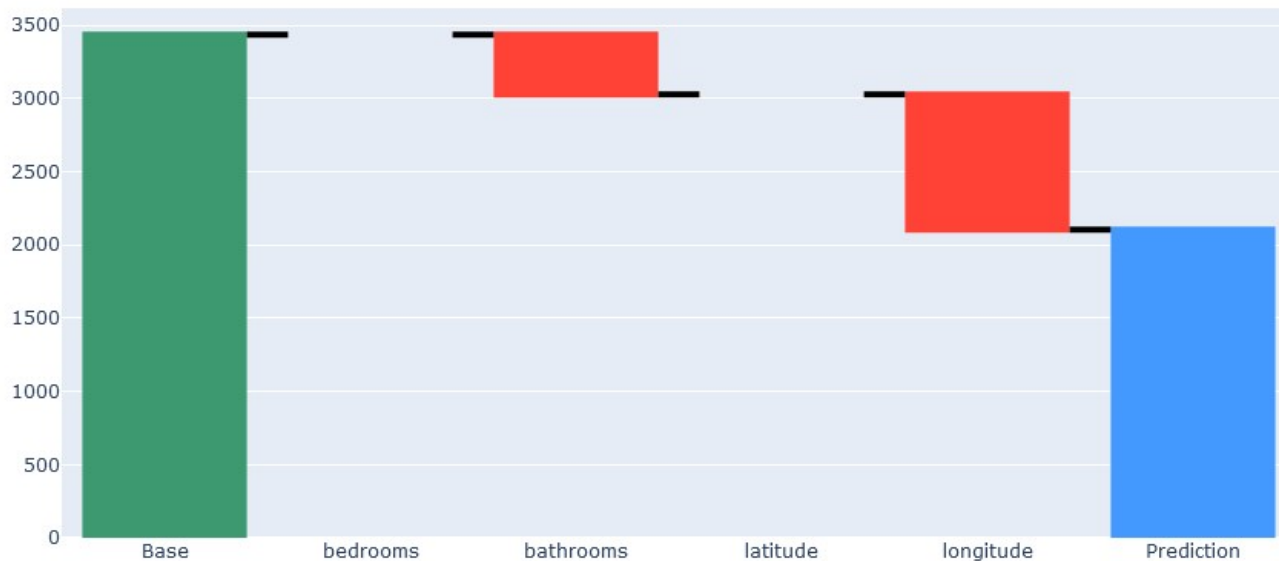
Prediction : 2607.8296004300305



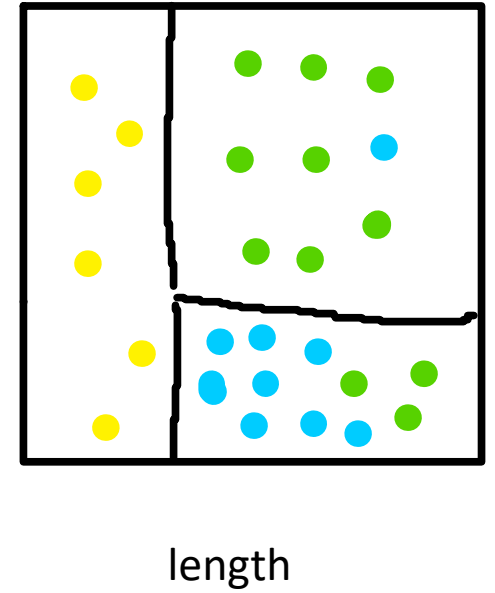
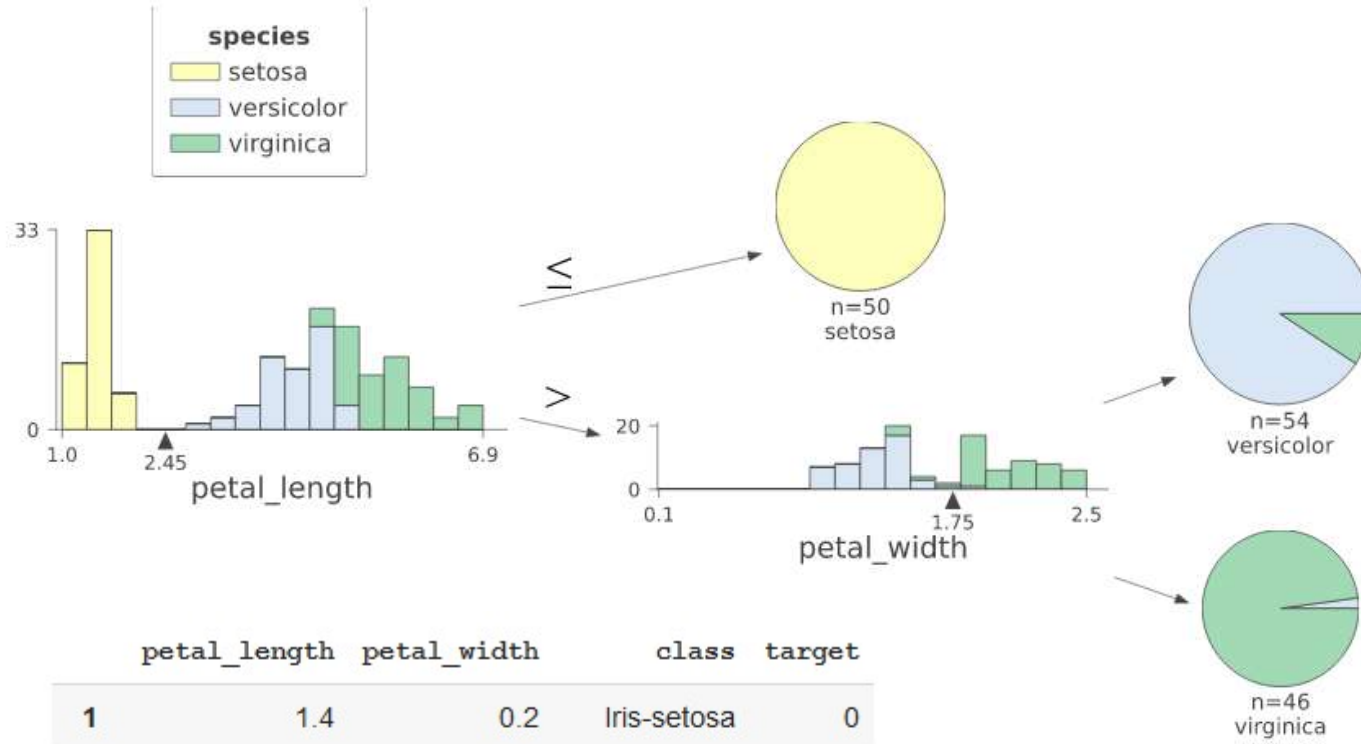
Donnerstag, 1. Juli 2021 17:45



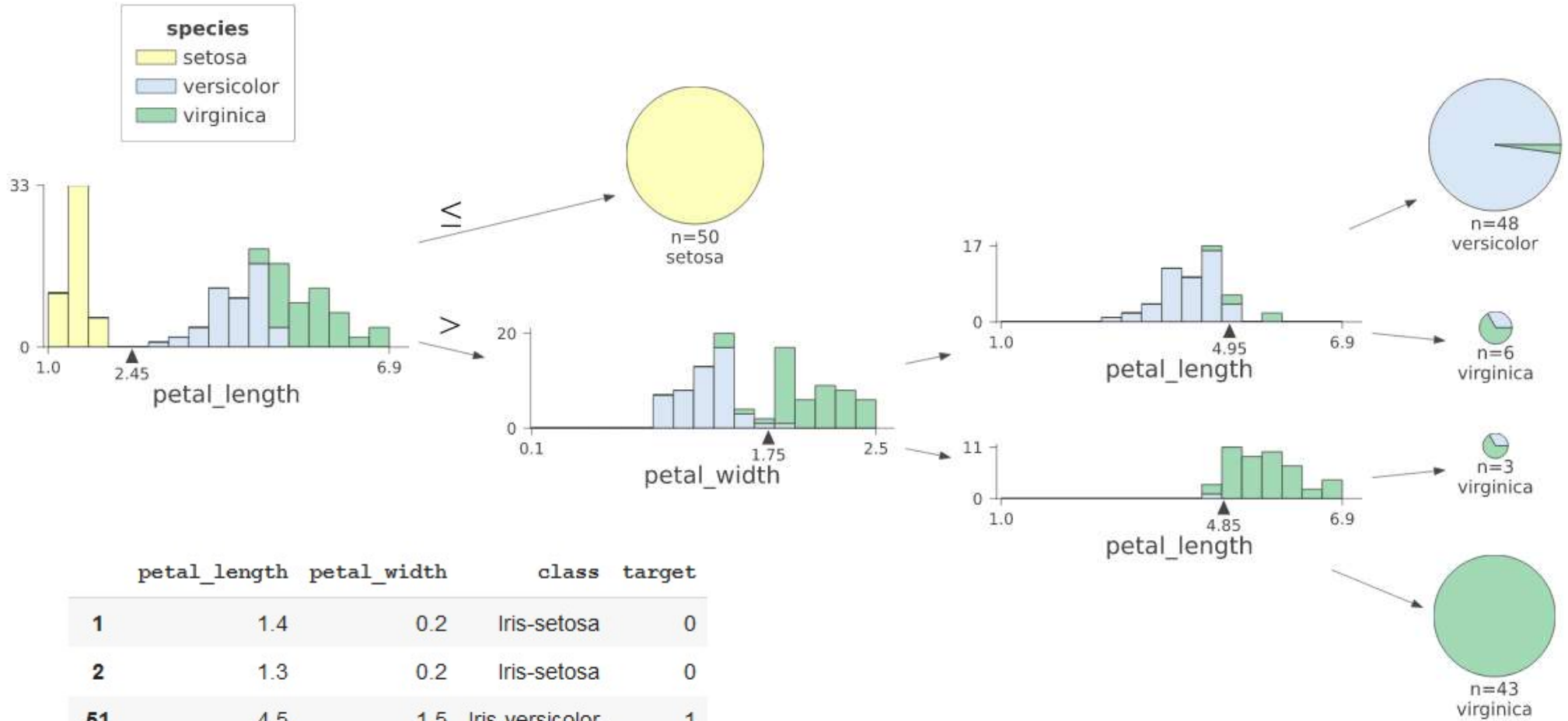
Prediction : 2104.545399698341



Tree depth 2



Tree depth 3



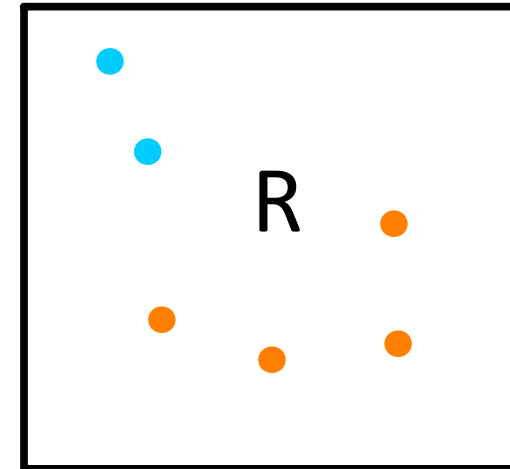
	petal_length	petal_width	class	target
1	1.4	0.2	Iris-setosa	0
2	1.3	0.2	Iris-setosa	0
51	4.5	1.5	Iris-versicolor	1
52	4.9	1.5	Iris-versicolor	1
101	5.1	1.9	Iris-virginica	2
102	5.9	2.1	Iris-virginica	2

$$\text{Gini}(R) = 1 - \sum_{c \in C} \text{Prob}(c, R)^2$$

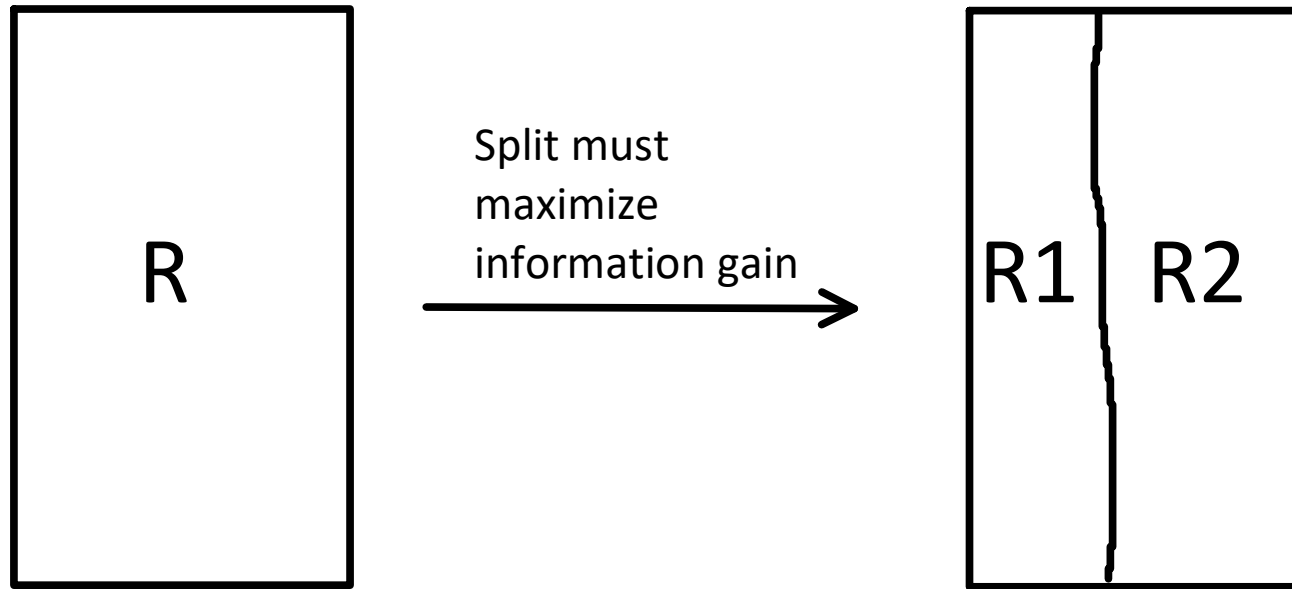
R	Region
Prob	Probability
Gini	Gini Value
C	Set of all classes
c	Single class

Class tags:

- Class1: orange
- Class2: blue



$$\begin{aligned} \text{Gini}(R) &= 1 - \left(\left(\frac{2}{6} \right)^2 + \left(\frac{4}{6} \right)^2 \right) \\ &= 1 - \left(\left(\frac{1}{3} \right)^2 + \left(\frac{2}{3} \right)^2 \right) \\ &= 1 - \left(\frac{1}{9} + \frac{4}{9} \right) \\ &= 1 - \frac{5}{9} \\ &= \frac{4}{9} \end{aligned}$$



$$\text{InformationGain} = \text{Gini}(R) - \left(\frac{1}{\text{count}(R_1)} \text{Gini}(R_1) + \frac{1}{\text{count}(R_2)} \text{Gini}(R_2) \right)$$

See:

<https://towardsdatascience.com/decision-tree-an-algorithm-that-works-like-the-human-brain-8bc0652f1fc6>

Problems of trees

- Too specific
- Overfit on trainings data

Assume leafs only contain one record

- Trainings error would get zero

Train many tree models

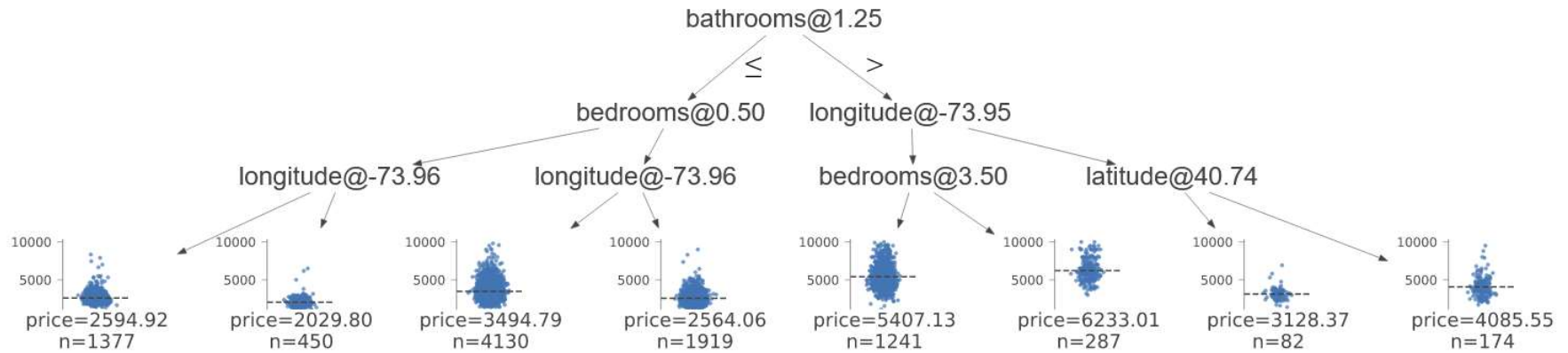
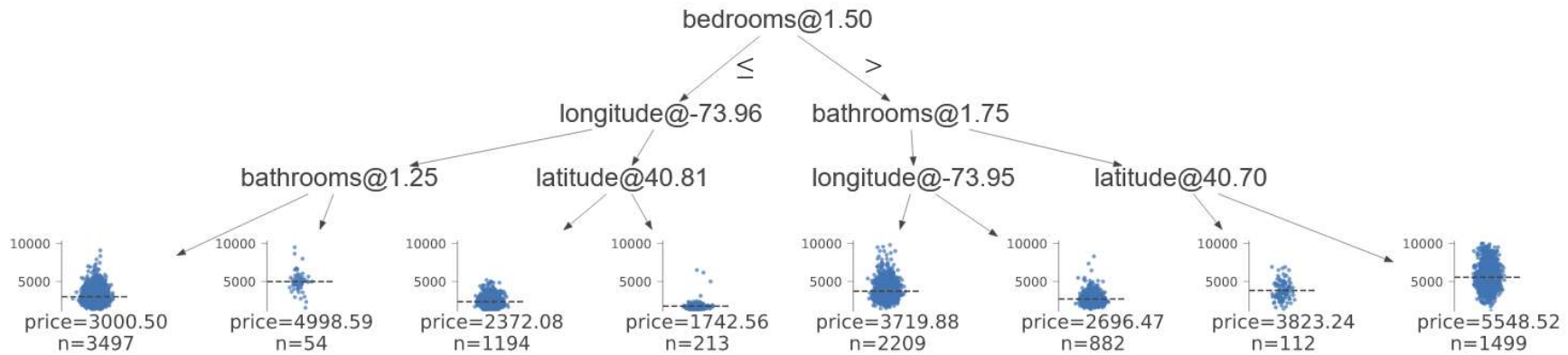
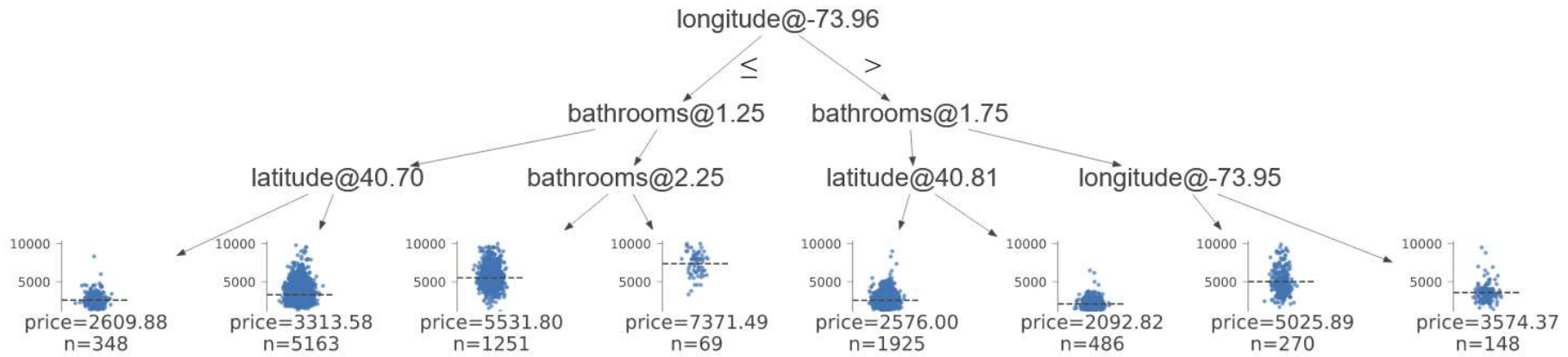
- Randomly select subset of training data (with replacement, bootstrapping)
- Randomly select subset of features

Prediction - Regression

- Ask all trees for prediction
- Average results
- Two levels of averaging
 - Leaf level of each singular tree
 - Average of all trees

Prediction - Classification

- Ask all trees for prediction
- Majority vote
- Two levels of voting
 - Leaf level of each singular tree
 - Combining votes of all trees



Permutation Importance

